Signal Transient and Crosstalk Model of Capacitively and Inductively Coupled VLSI Interconnect Lines

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Abstract - Analytical compact form models for the signal transients and crosstalk noise of inductive-effect-prominent multi-coupled RLC lines are developed. Capacitive and inductive coupling effects are investigated and formulated in terms of the equivalent transmission line model and transmission line parameters for fundamental modes. The signal transients and crosstalk noise expressions of two coupled lines are derived by using a wave-form approximation technique. It is shown that the models have excellent agreement with SPICE simulation.

Keywords: Crosstalk, inductance effect, interconnect lines, signal transient, transmission lines.

1 Introduction

As the clock frequency of VLSI circuits dramatically increases over several GHz, interconnect lines play a pivotal role in the determination of circuit performance [1], [2]. In the strongly-coupled global interconnect lines, the inductances of the lines have significant effects on the circuits’ transient characteristics and coupling noises [3]. In such high-speed integrated circuits, since the signal integrity concerned with interconnect lines cannot be guaranteed without taking inductance-prominent transmission line effects into account, the RLC (resistive, inductive, and capacitive) transmission line model for the accurate signal integrity verification of the circuits becomes essential.

Since the electromagnetic coupling effects crucially limit integrated circuit performance, they have to be considered in the early phase of circuit design. Up to now, numerous crosstalk models have been developed [4]-[8]. Traditionally, most of the crosstalk models for the on-chip interconnect lines are based on capacitive coupling effect [4], [5]. It is simply because the RC seems to be more prominent than the “L/R.” However, as the circuit switching speed increases, the inductive coupling effect plays an important role. For on-chip interconnect lines, the signal transients of the coupling-aware switching lines and the crosstalk noise of a quiet line cannot be accurately determined only with the capacitive coupling model [9].

Recently, there has been a lot of research concerned with multi-coupled RLC line analysis and modeling [6]-[8]. However, unlike the capacitance-effect-prominent lines (i.e., RC lines), a compact form of CAD model concerned with the signal transients of inductive-effect-prominent coupled lines may not be readily determined since the signal transients of inductance-effect-prominent lines result in complicatedly oscillating non-monotonic wave-shapes.

In this work, assuming (1) linear driver model (i.e., voltage source and source resistance), (2) identical RLC coupled lines, and (3) step input signals, analytical models for signal transients and crosstalk noise of coupled RLC lines are developed and verified. The paper is organized as follows. First, the fundamental modes of the coupled lines and the equivalent transmission line parameters corresponding to the fundamental modes are determined. Next, analytical forms of signal transient expressions for the fundamental modes are developed by using a waveform approximation technique [10]. Finally, it is shown that the developed models have excellent agreement with SPICE simulation.

2 Fundamental Modes of Coupled Interconnect Lines

In the frequency domain, the n-coupled transmission line equations are given by [11],

\[
\frac{d^2}{dz^2}[V(z)] = [Z][Y][V(z)],
\]

\[
\frac{d^2}{dz^2}[I(z)] = [Y][Z][I(z)],
\]

where

\[
[Z] = [R] + j\omega[L],
\]

\[
[Y] = [C] + j\omega[L] = j\omega[C].
\]

[R], [L], [C], and [G] are PUL (per unit length)-transmission line parameter matrices and are
The per unit length (PUL) transmission line parameters for the identical two coupled lines are defined as

\[ R_{\text{PUL}} = \begin{bmatrix} R & 0 \\ 0 & R \end{bmatrix}, \]

\[ L_{\text{PUL}} = \begin{bmatrix} L_{11} & L_{12} \\ L_{21} & L_{22} \end{bmatrix}, \]

\[ C_{\text{PUL}} = \begin{bmatrix} (C_{11} + C_{12}) & -C_{12} \\ -C_{12} & (C_{11} + C_{12}) \end{bmatrix}. \]

In this case, there exist two eigen modes, even and odd mode [10]. Physically, the even mode is a switching mode in which both lines switch in the same direction (i.e., ↑↑) and the odd mode means a switching mode in which lines switch in opposite directions (i.e., ↑↓) [6]. All the switching patterns of two coupled lines are represented by linear combination of those modes. Then coupled lines can be converted to an equivalent single line as shown in Figure 1 and 2. The equivalent transmission line parameters for both the even mode and the odd mode are

\[ L_{\text{even}} = L_{11} + L_{12}, \quad C_{\text{even}} = C_{11}, \]

\[ L_{\text{odd}} = L_{11} - L_{12}, \quad C_{\text{odd}} = C_{11} + 2C_{12}. \]

Thus, the characteristic impedances and propagation constants of the even and odd mode can be determined as [9]

\[ Z_{\text{even}} = \frac{R + jo\omega (L_{11} + L_{12})}{joC_{11}}, \]

\[ Z_{\text{odd}} = \frac{R + jo\omega (L_{11} - L_{12})}{jo(C_{11} + 2C_{12})}, \]

\[ \gamma_{\text{even}} = \sqrt{R + jo\omega (L_{11} + L_{12})joC_{11}}, \]

\[ \gamma_{\text{odd}} = \sqrt{R + jo\omega (L_{11} - L_{12})jo(C_{11} + 2C_{12})}. \]

Note, any signal traveling in the coupled transmission line system can be expressed as the linear combination of these eigen modes. For example, if the first line is switching from logic 0 to logic 1 and the second line is in a quiet state, the signals on the respective lines due to switching (i.e., ↑0) can be readily determined by using a symbolic operation

\[ \begin{pmatrix} \uparrow \downarrow \end{pmatrix}_{\text{even}} = \frac{1}{2}(\uparrow \downarrow + \uparrow \downarrow) = \frac{1}{2}((\text{even}) + (\text{odd})) \]

\[ \Rightarrow \text{the switching signal transient of two coupled lines.} \]

\[ \begin{pmatrix} \uparrow \downarrow \end{pmatrix}_{\text{odd}} = \frac{1}{2}(\uparrow \downarrow - \uparrow \downarrow) = \frac{1}{2}((\text{even}) - (\text{odd})) \]

\[ \Rightarrow \text{the cross-talk noise of two coupled lines.} \]

Note, the line of interest for the switching mode is marked with a square box.

![Figure 1. Two coupled transmission line circuit model](image1)

![Figure 2. Equivalent transmission line circuit model for two coupled lines](image2)

### 3 Signal Transient and Crosstalk Models

All the switching patterns of the two coupled lines can be represented by using two fundamental switching modes (i.e., two eigen modes), so called, even mode (↑↑) and odd mode (↑↓). If the line driver is modeled with the unit step signal and source resistance while the load is modeled as a capacitance, signal transient and crosstalk noise can be mathematically formulated by using the traveling-wave-based waveform approximation technique (TWA)-based signal transient expressions [10].

For on-chip interconnects, in the odd mode, the relation \( R >> \omega L_{\text{odd}} \) is generally verified [9]. Thus, for simplicity, the odd mode waveform \( V_{\text{odd}}(t) \) can be approximately determined only with 3-poles.

\[ V_{\text{odd}}(t) \quad (0 \leq t \leq \infty) \approx \frac{V_{\text{odd}}(0)}{\delta_{\text{odd}}(t)}, \]

The subscript “3pole” indicates the three-pole-based time-domain response [10]. The time of flight \( t_{\text{f0}} \) and a time difference with the wave reflection \( \delta_{\text{even}} \) for the even mode are defined as

\[ t_{\text{f0}} = \sqrt{\frac{1}{2}L_{\text{even}}(1 - C_{\text{even}}^2 + C_3^2)}, \]

\[ \delta_{\text{even}} = \sqrt{\frac{1}{2}L_{\text{even}}(1 - C_{\text{even}}^2 + C_3^2) - \frac{1}{2}L_{\text{even}}C_{\text{even}}}. \]

Thus, for the even mode, the waveform can be determined as

\[ V_{\text{even}}(t) \quad (0 \leq t < (t_{\text{f0}} - \delta_{\text{even}})) = 0, \]

\[ V_{\text{even}}(t) = V_{\text{even}}(t_{\text{f0}} - \delta_{\text{even}}) \leq t \leq (t_{\text{f0}} + \delta_{\text{even}}) \]

\[ V_{\text{even}}(t) = \frac{V_{\text{even}}(0)}{\delta_{\text{even}}(t)}[(t - \delta_{\text{even}})^2 + V_{\text{odd}}(t_{\text{f0}})], \]

\[ V_{\text{even}}(t) = \frac{V_{\text{even}}(0)}{\delta_{\text{even}}(t)}[(3t_{\text{f0}} - \delta_{\text{even}})], \]

\[ V_{\text{even}}(t) = \frac{V_{\text{even}}(0)}{\delta_{\text{even}}(t)}[(3t_{\text{f0}} - \delta_{\text{even}})]. \]
\[ v_{\text{trans}} = 2v_{\text{trans}}(t_f^{\text{trans}}) + w_{\text{even}} \left[ 1 - \exp \left( - \frac{t - (t_f^{\text{trans}} + \delta_{\text{trans}})}{t_{\text{even}}} \right) \right], \quad (21) \]

where
\[ \begin{align*}
    t_{\text{even}} &= (l \cdot R)(l \cdot c_{\text{even}} + C_2), \quad (22) \\
    w_{\text{even}} &= \frac{v_{\text{trans}}(3t_f^{\text{trans}} - \delta_{\text{trans}}) - 2v_{\text{trans}}(t_f^{\text{trans}})}{1 - \exp \left( - \frac{2(t_f^{\text{trans}} - \delta_{\text{trans}})}{t_{\text{even}}} \right)}, \quad (23)
\end{align*} \]

Since the far end reflection wave after three time of flight may be not significant, the even mode can be approximated as
\[ v_{\text{even}}(t \geq (3 \cdot t_f^{\text{trans}} - \delta_{\text{trans}})) \approx v_{\text{trans}}(t \geq (3 \cdot t_f^{\text{trans}} - \delta_{\text{trans}})). \quad (24) \]

### 4 Verification of the Models

In order to verify the accuracy of the proposed model, transmission line parameters are determined for test structures (see Figure 3) by using a commercial field solver. Test structures are based on a DSM (deep submicron) technology [1], [12]. The transmission line parameters for two coupled lines are
\[
\begin{align*}
    [R] &= \begin{bmatrix} 44.44 & 0 \\ 0 & 44.44 \end{bmatrix} \text{Ω}, \\
    [L] &= \begin{bmatrix} 0.612 & 0.380 \\ 0.380 & 0.612 \end{bmatrix} \text{nH}, \\
    [C] &= \begin{bmatrix} 161.83 & -54.884 \\ -54.884 & 161.83 \end{bmatrix} \text{fF}.
\end{align*}
\]

As shown in Figure 4, signal transients and crosstalk noise using the proposed compact models are compared with the SPICE simulation. The proposed models have excellent agreement with SPICE simulation.

### 5 Conclusion

In this work, compact analytical models for the signal transients and crosstalk noises of coupled RLC transmission lines are developed. The coupled lines are decoupled with fundamental modes and then their signal transients and crosstalk noise are determined by using "traveling-wave-based waveform approximation" which is a kind of closed-form model. Since a switching pattern for the coupled lines can be readily decomposed with the fundamental modes, the signal transients and crosstalk can be very efficiently determined. It was shown that the developed compact models have excellent agreement with SPICE simulation.
References


