Discontinuous RC Interconnect Line Analysis for Accurate Timing Determination

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Abstract—In this research, discontinuous interconnect lines are modeled as a cascaded line composed of many uniform interconnect lines. The system functions of respective uniform interconnect lines are determined, followed by its time domain response. Since the time domain response expression is a transcendental form, the waveform expression is reconfigured as an approximated linear expression. The proposed model has less than 2% error in the delay estimation.

Keywords-Interconnect modeling, RC interconnects.

I. INTRODUCTION

As the clock frequency of VLSI circuits dramatically increases over several GHz, interconnect lines play a pivotal role in the determination of circuit performance [1]-[3]. Thus, signal integrity of the interconnect lines has to be verified in the early stage of circuit design.

In practical integrated circuits, most of interconnect lines is neither simple uniform nor isolated straight lines but discontinuous lines. That is, the characteristic impedance of the line of the interest may be changed during the signal propagation from a source to a destination because of neighbor lines. Physically, while signal paths near circuit blocks may have narrow routing spaces, the other area may be more enough routing space. Consequently, a signal path may have different line width for suitable layouts. Alternatively, many of the signal lines may be complicatedly coupled with neighbor lines in parts. As an example, an interconnect line from a circuit block to another circuit block may be coupled in a part as shown in Fig. 1. In such case, the signal transient of the line may be significantly affected by neighbor line switching. That means a part of the line may have different characteristics with the remaining part of the line. Thus, existing simple timing models that assume a uniform straight line may not be accurate enough as shown in Fig. 2. In reality, although such complicated line can be accurately evaluated with SPICE simulation, it may not be suitable for the timing verification of practical circuits which has myriad interconnect nets. Thus, a much simpler timing model for such lines is highly required to be incorporated into various circuit design CAD tools.

The authors of [4] derived a simple timing model for tree-structure-like discontinuous lines. However, the model is not accurate enough since they aggressively approximate the expression during the model derivation. Thus, it needs to be
II. TIME DOMAIN RESPONSE OF DISCONTINUOUS INTERCONNECT LINE

A. System Function of RC Interconnect Line

In [5], assuming an one dominant pole, a step input response for an RC interconnect line of Fig. 3 can be represented as

\[ v_i(t) = 1 + K \exp(-t/\tau), \]  

where

\[ K = -1.01(R_2 + C_\tau \pi/4), \]  

\[ \tau = RC(R_2C_x + R_2 + C_x + (2/\pi)^2)/1.04, \]  

\[ R_2 = R_x/R \cdot C_2 = C_x/C. \]

The total resistance (capacitance) of the line. It can be represented with \( R = R_x \cdot j/(C = C_x \cdot j) \), where \( R_x \) and \( C_x \) are per-unit length (PUL)-resistance and PUL-capacitance, respectively and \( j \) is the length of the line. Since the frequency domain response of (1) becomes

\[ V_i(s) = \frac{s(1 + K)x + 1}{s(\tau x + 1)}, \]

the system function of the line can be regarded as

\[ H(s) = \frac{s(1 + K)x + 1}{s(\tau x + 1)} = H'(s) + (1 + K)H''(s). \]  

where

\[ H'(s) = \frac{1}{(\tau x + 1), \]  

\[ H''(s) = \frac{\tau x}{(\tau x + 1). \]

In distributed RC line, the effective time constant is much smaller than that of the lumped model. Thus, defining \( R \) as the total resistance of the system, \( C \) can be determined as

\[ R' = R_x + R = R(R_x + 1), \]  

\[ C' = \tau / R'. \]  

Thus, since the time constant becomes

\[ \tau = R'C', \]

\[ H'(s) \text{ and } H''(s) \text{ can be rewritten as} \]

\[ H'(s) = \frac{1}{(sR'C')}, \]  

\[ H''(s) = \frac{aR'C'}{(sR'C')^2 + 1). \]  

Therefore, (3) can be represented with the combination of two equivalent circuits as shown in Fig. 4.

B. Time Domain Response at Intermediate Node

A discontinuous line can be represented with distributed circuit model as shown in Fig. 5(a). Since the line is discontinuous, a system function needs to be determined in the discontinuous node (e.g., node-1). Note, unlike the circuit of Fig. 3 that has the lumped capacitance load, the intermediate node (i.e., the node-1) of the circuit of Fig. 5(a) has the interconnect line as a load. Thus, (3) has to be modified a bit.

In order to determine the system function of the node-1, the driving point impedance of the right hand side of the node is represented with an approximated lumped circuit as shown in Fig. 5(a) [6]. The lumped circuit model parameters can be determined as

\[ R_x = y_1^2/y_2^2, \]  

\[ C_x = y_2^2/y_2, \]  

\[ C_{\text{n}} = y_1 - y_2^2/y_2. \]

where \( y_1, y_2, \) and \( y_3 \) are the moments of the admittance for node-1. Note, as shown in Fig. 6, the time domain response for such an approximation circuit has excellent agreement with that of the original circuit model.

Regarding \( C_n \) as the load of the left-hand side of the node-1, the discontinuous line can be represented as in Fig. 5(b). Note, \( R_x \) and \( C_x \) can be determined by using the similar technique as in determining \( R' \) and \( C' \),

\[ R_x \equiv R_x + R = R(R_x + 1), \]  

\[ C_x \equiv \tau / R. \]  

Since the circuit of Fig. 5 (b) is similar to that of Fig. 4, combining (3) with (6), the transfer function of the left-hand-side line can be represented as

\[ H_i(s) \approx H_i'(s) + (1 + K_i)H_i''(s), \]

where

\[ H_i'(s) = \frac{1 + sR_xC_x}{1 + s[R_x(C_n + C_x) + R_xC_x] + s' R_x C_x C_{\text{n}}.} \]
Thus, \( V_1(s) \) becomes
\[
V_1(s) = \frac{1}{s} H_2(s) = \frac{1}{s} \frac{1}{R_s R_a C_1 C_3} \frac{1}{s^2 R_s R_a C_1 C_3 + s (s - p_1) (s - p_2)}.
\]
(10)

where
\[
p_{1,2} = -b \pm \sqrt{b^2 - 4a},
\]
\[
a = R_s R_a C_1 C_3, \quad b = R_s (C_2 + C_3) + R_a C_1.
\]

Thus, the time domain counterpart part is
\[
v_1(t) = 1 + A_1 e^{p_1 t} + A_2 e^{p_2 t},
\]
(11)

C. **Time Domain Response at Output Node**

Since \( H_2(s) \) was determined, only if \( H_3(s) \) is known, the response of the discontinuous line \( V_3(s) \) can be readily determined from the circuit of Fig. 5(a),
\[
V_3(s) = \frac{1}{s} \left[ H_3(s) \right] \left[ (1 + K_a) H_3(s) \right] H_3(s).
\]
(12)

Since the circuit of Fig. 5 (b) is similar to that of Fig. 4, from (1) and (3), \( H_3(s) \) can be determined as shown below
\[
H_3(s) = \frac{\tau_2 (1 + K_a) s + 1}{(\tau_2 s + 1)}.
\]
(13)

Thus, from (3), (8), (9), and (10), \( V_3(s) \) can be rewritten as
\[
V_3(s) = \frac{1}{a} \frac{1}{s^2 - p_1 (s - p_2)} \frac{1}{s^2 - p_2}.
\]
(14)

The time domain response becomes
\[
v_3(t) = 1 + B_1 e^{p_1 t} + B_2 e^{p_2 t}.
\]
(15)

III. **Model Verification**

In order to verify the proposed technique, the various interconnect line parameters are defined as summarized in Table I. Then signal transients using (19) are compared with SPICE simulation. As shown in Fig. 7, the signal transient wave shapes obtained using the proposed model have excellent agreement with SPICE simulation.

The 50% time delay and 90% time delay for various line parameters are determined as summarized in Table II. Note,
the error between the model and SPICE simulation is less than 2%. Since the wave shapes are in agreement with the analytical model, interconnect line timing data may be readily determined by using the same algorithm [7].

IV. CONCLUSION

In this research, since the generic circuit model of a discontinuous line is too much complicated to be analyzed, it was modeled as a cascaded interconnects line chain. Then an analytical expression for the signal transient wave shapes was derived by using the new simple equivalent circuit. It was shown that the model expression has excellent agreement with SPICE simulation in less than 2% error in the delay estimation.

### TABLE I. TESTED TRANSMISSION LINE PARAMETERS.

<table>
<thead>
<tr>
<th>Case</th>
<th>$R_1$ (Ω)</th>
<th>$C_1$ (pF)</th>
<th>$R_2$ (Ω)</th>
<th>$C_2$ (pF)</th>
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</thead>
<tbody>
<tr>
<td>Case 1</td>
<td>30</td>
<td>1</td>
<td>100</td>
<td>0.2</td>
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<tr>
<td>Case 2</td>
<td>100</td>
<td>0.2</td>
<td>200</td>
<td>0.3</td>
</tr>
<tr>
<td>Case 3</td>
<td>50</td>
<td>0.1</td>
<td>300</td>
<td>0.6</td>
</tr>
<tr>
<td>Case 4</td>
<td>200</td>
<td>0.1</td>
<td>50</td>
<td>1</td>
</tr>
</tbody>
</table>

![Graph](image)

Figure 7. Comparison of voltage waveforms at the output node. $R_s=50Ω$, $C_L=0.2pF$.

### TABLE II. TIME DELAY COMPARISONS OF THE MODEL AND SPICE SIMULATION FOR VARIOUS SOURCES AND LOADS.

<table>
<thead>
<tr>
<th>$R_s$ (Ω)</th>
<th>$C_L$ (pF)</th>
<th># of case</th>
<th>50% Delay</th>
<th>90% Delay</th>
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</thead>
<tbody>
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<td>99.2</td>
<td>98.4</td>
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<td>2</td>
<td>105.8</td>
<td>105.4</td>
</tr>
<tr>
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<td></td>
<td>3</td>
<td>160.8</td>
<td>161.6</td>
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<td>114.8</td>
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<td>3</td>
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<td>288.5</td>
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REFERENCES